



Technical Note

The interaction of thermal nonequilibrium and heterogeneous conductivity effects in forced convection in layered porous channels

D.A. Nield^a, A.V. Kuznetsov^{b,*}^a Department of Engineering Science, University of Auckland, Auckland, New Zealand^b Department of Mechanical and Aerospace Engineering, North Carolina State University, Campus Box 7910, Raleigh, NC 27695-7910, USA

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Abstract

Forced convection in a parallel plate channel filled with a porous medium, consisting of two layers with the same porosity and permeability but with different solid conductivity, and saturated by a single fluid, is analyzed using a two-temperature model. It is found that the effect of local thermal nonequilibrium is particularly significant when the solid conductivity in each layer is greater than the fluid conductivity, and in these circumstances the effect is to reduce the Nusselt number defined in terms of the overall effective thermal conductivity. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Owing to the use of hyperporous media in the cooling of electronic equipment, there has recently been renewed interest in the classical problem of forced convection in a porous medium channel or duct. Global heterogeneity effects due to the variation of permeability and/or thermal conductivity, with the medium layered in the transverse direction, have been analyzed in a number of papers by the authors [1–4]. These papers have been concerned with the usual situation in which thermal equilibrium between the solid and fluid phases can be assumed. However, there are several industrial applications where high-speed flow leads to a significant degree of local thermal nonequilibrium (LTNE), even in the case of steady forced convection, and this situation for a homogeneous porous medium has also been analyzed by the authors [5,6]. In the present paper the interaction

between heterogeneity effects and LTNE effects is investigated.

2. Analysis

We consider the case (see Fig. 1(a)) of a parallel plate channel with walls at $y^* = \pm H$, divided into core and sheath layers occupied, respectively, by two porous media, of the same porosity ϕ and permeability K , and saturated by the same fluid of thermal conductivity k_f , but with the solid thermal conductivity k_s being given by

$$k_s = k_{s1} \quad \text{for } 0 < |y^*| < \xi H, \quad (1a)$$

$$k_s = k_{s2} \quad \text{for } \xi H < |y^*| < H. \quad (1b)$$

Asterisks are used to denote dimensional variables. Thus the mean solid conductivity is

$$\bar{k}_s = \xi k_{s1} + (1 - \xi)k_{s2}. \quad (2)$$

We define

$$\bar{k}_{s1} = k_{s1}/\bar{k}_s, \quad \bar{k}_{s2} = k_{s2}/\bar{k}_s, \quad k_{rel} = \bar{k}_s/k_f, \quad (3a)$$

* Corresponding author. Tel.: +1-919-515-5292; fax: +1-919-515-7968.

E-mail address: avkuznet@eos.ncsu.edu (A.V. Kuznetsov).

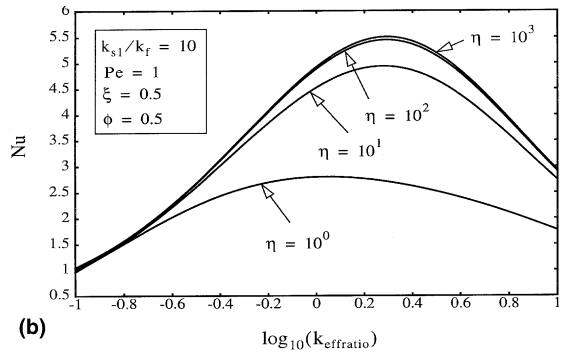
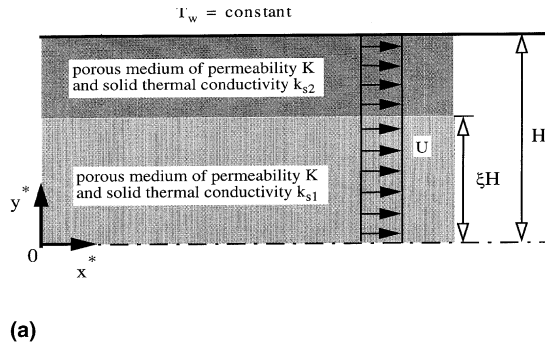


Fig. 1. Definition sketch, and plots of Nusselt number Nu versus effective conductivity ratio k_{effratio} for various values of the solid-to-fluid heat transfer number η ($\xi = 0.5, \phi = 0.5$).

$$k_{\text{eff}} = \phi k_f + (1 - \phi) \bar{k}_s, \tag{3b}$$

$$k_{\text{effratio}} = \frac{\phi k_f + (1 - \phi) k_{s2}}{\phi k_f + (1 - \phi) k_{s1}}.$$

The thermal energy equations for the solid and fluid phases are taken to be

$$(1 - \phi) \nabla^* \cdot (k_s \nabla^* T_s^*) + h_{fs} (T_f^* - T_s^*) = 0, \tag{4a}$$

$$\phi \nabla^* \cdot (k_f \nabla^* T_f^*) + h_{fs} (T_s^* - T_f^*) = (\rho c_p)_f \mathbf{v} \cdot \nabla^* T_f^*. \tag{4b}$$

The fluid–solid heat exchange coefficient h_{fs} is assumed to be uniform. The case where the wall temperature T_w is uniform is considered. For the fluid flow, Darcy’s law is assumed. Since the permeability is uniform, so is the axial velocity U .

We introduce dimensionless variables and parameters by

$$x = \frac{x^*}{H}, \quad y = \frac{y^*}{H}, \quad \theta_f = \frac{T_f^* - T_w}{T_{\text{ref}}}, \quad \theta_s = \frac{T_s^* - T_w}{T_{\text{ref}}}, \tag{5}$$

$$Pe = \frac{UH(\rho c_p)_f}{k_f}, \quad \eta = \frac{h_{fs} H^2}{k_{\text{eff}}}. \tag{6}$$

Thus Pe is the Péclet number based on fluid properties, and η is the dimensionless exchange parameter introduced by Nield and Kuznetsov [6].

It is convenient to carry out the analysis in terms of the following parameters:

$$N_f = \frac{\phi}{Pe}, \quad N_s = \frac{(1 - \phi) k_{\text{rel}}}{Pe}, \tag{7}$$

$$N_h = \frac{\eta [\phi + (1 - \phi) k_{\text{rel}}]}{Pe}.$$

With subscripts 1, 2 referring to the subdomains $0 < y < \xi, \xi < y < 1$, respectively, then, for $j = 1, 2$, we have

$$\left[N_s \tilde{k}_{sj} \frac{\partial^2}{\partial y^2} - N_h \right] \theta_{sj} + N_h \theta_{fj} = 0, \tag{8a}$$

$$N_h \theta_{sj} + \left[N_f \frac{\partial^2}{\partial y^2} - N_h - \frac{\partial}{\partial x} \right] \theta_{fj} = 0. \tag{8b}$$

These equations constitute an eighth-order system, which is to be solved subject to the following eight symmetry/matching/boundary conditions:

$$\frac{\partial \theta_{f1}}{\partial y} = \frac{\partial \theta_{s1}}{\partial y} = 0 \quad \text{at } y = 0, \tag{9}$$

$$\theta_{f1} = \theta_{f2}, \quad \theta_{s1} = \theta_{s2}, \quad \frac{\partial \theta_{f1}}{\partial y} = \frac{\partial \theta_{f2}}{\partial y}, \tag{10}$$

$$\tilde{k}_{s1} \frac{\partial \theta_{s1}}{\partial y} = \tilde{k}_{s2} \frac{\partial \theta_{s2}}{\partial y} \quad \text{at } y = \xi, \tag{11}$$

$$\theta_{f2} = \theta_{s2} = 0 \quad \text{at } y = 1.$$

The individual equations in Eq. (10) express the matching of temperature and heat flux at the interface between the two layers, for the fluid and solid phases separately. (This matching is based on the uniformity principle discussed by Nield and Kuznetsov [6], that requires that the matching conditions are independent of the porosity.)

The variables are now separated by letting

$$\theta_{fj} = \Theta_{fj} e^{\lambda x}, \quad \theta_{sj} = \Theta_{sj} e^{\lambda x}. \tag{12}$$

With D denoting d/dy , we then obtain

$$\left[N_s \tilde{k}_{sj} D^2 - N_h \right] \Theta_{sj} + N_h \Theta_{fj} = 0, \tag{13a}$$

$$N_h \Theta_{sj} + [N_f D^2 - N_h - \lambda] \Theta_{fj} = 0. \tag{13b}$$

Elimination of Θ_{sj} leads to

$$\left\{ \left(N_s \tilde{k}_{sj} D^2 - N_h \right) (N_f D^2 - N_h - \lambda) - N_h^2 \right\} \Theta_{fj} = 0. \tag{14}$$

The solution of this equation subject to the conditions in Eqs. (9) and (11) is

$$\Theta_{f1} = A \cos s_{21} y + B \cosh s_{\beta 1} y, \tag{15a}$$

$$\Theta_{f2} = C \sin s_{22} (1 - y) + D \sinh s_{\beta 2} (1 - y). \tag{15b}$$

Here, for $j = 1, 2$,

$$s_{2j} = \left\{ \left[- \left(N_h N_f + N_h N_s \tilde{k}_{sj} + \lambda N_s \tilde{k}_{sj} \right) + \left(N_h N_f + N_h N_s \tilde{k}_{sj} + \lambda N_s \tilde{k}_{sj} \right)^2 - 4 \lambda N_h N_f N_s \tilde{k}_{sj} \right]^{1/2} \right\} / \left\{ 2 N_f N_s \tilde{k}_{sj} \right\}^{1/2}, \quad (16a)$$

$$s_{\beta j} = \left\{ \left[N_h N_f + N_h N_s \tilde{k}_{sj} + \lambda N_s \tilde{k}_{sj} + \left(N_h N_f + N_h N_s \tilde{k}_{sj} + \lambda N_s \tilde{k}_{sj} \right)^2 - 4 \lambda N_h N_f N_s \tilde{k}_{sj} \right]^{1/2} \right\} / \left\{ 2 N_f N_s \tilde{k}_{sj} \right\}^{1/2}. \quad (16b)$$

The matching conditions in Eq. (10) then imply that

$$Mc = 0, \quad (17)$$

where

M

$$= \begin{bmatrix} C_1 & C_2 & -S_3 & -S_4 \\ -s_{\alpha 1} S_1 & s_{\beta 1} S_2 & s_{\alpha 2} C_3 & s_{\beta 2} C_4 \\ -s_{\alpha 1}^2 C_1 & s_{\beta 1}^2 C_2 & s_{\alpha 2}^2 S_3 & -s_{\beta 2}^2 S_4 \\ \frac{-\tilde{k}_{s1} s_{\alpha 1} S_1}{N_s \tilde{k}_{s1} s_{\alpha 1}^2 + N_h} & \frac{\tilde{k}_{s1} s_{\beta 1} S_2}{-N_s \tilde{k}_{s1} s_{\beta 1}^2 + N_h} & \frac{\tilde{k}_{s2} s_{\alpha 2} C_3}{N_s \tilde{k}_{s2} s_{\alpha 2}^2 + N_h} & \frac{\tilde{k}_{s2} s_{\beta 2} C_4}{-N_s \tilde{k}_{s2} s_{\beta 2}^2 + N_h} \end{bmatrix},$$

and $c = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$, (18)

where in turn

$$\begin{aligned} C_1 &= \cos s_{\alpha 1} \zeta, & S_1 &= \sin s_{\alpha 1} \zeta, \\ C_2 &= \cosh s_{\beta 1} \zeta, & S_2 &= \sinh s_{\beta 1} \zeta, \\ C_3 &= \cos s_{\alpha 2} (1 - \zeta), & S_3 &= \sin s_{\alpha 2} (1 - \zeta), \\ C_4 &= \cosh s_{\beta 2} (1 - \zeta), & S_4 &= \sinh s_{\beta 2} (1 - \zeta). \end{aligned} \quad (19)$$

The eigenvalue equation (regarded as an equation for λ) is

$$\det M = 0. \quad (20)$$

With λ found, the corresponding eigenvector is given by

$$\frac{-A}{A_1} = \frac{B}{A_2} = \frac{-C}{A_3} = \frac{D}{A_4}, \quad (21)$$

where

$$A_1 = \begin{bmatrix} C_2 & -S_3 & -S_4 \\ s_{\beta 1} S_2 & s_{\alpha 2} C_3 & s_{\beta 2} C_4 \\ s_{\beta 1}^2 C_2 & s_{\alpha 2}^2 S_3 & -s_{\beta 2}^2 S_4 \end{bmatrix},$$

$$\begin{aligned} A_2 &= \begin{bmatrix} C_1 & -S_3 & -S_4 \\ -s_{\alpha 1} S_1 & s_{\alpha 2} C_3 & s_{\beta 2} C_4 \\ -s_{\alpha 1}^2 C_1 & s_{\alpha 2}^2 S_3 & -s_{\beta 2}^2 S_4 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} C_1 & C_2 & -S_4 \\ -s_{\alpha 1} S_1 & s_{\beta 1} S_2 & s_{\beta 2} C_4 \\ -s_{\alpha 1}^2 C_1 & s_{\beta 1}^2 C_2 & -s_{\beta 2}^2 S_4 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} C_1 & C_2 & -S_3 \\ -s_{\alpha 1} S_1 & s_{\beta 1} S_2 & s_{\alpha 2} C_3 \\ -s_{\alpha 1}^2 C_1 & s_{\beta 1}^2 C_2 & s_{\alpha 2}^2 S_3 \end{bmatrix}. \end{aligned} \quad (22)$$

(Without loss of generality one can put $D = 1$.)

Then Eqs. (13b), (15a) and (15b) yield

$$\Theta_{s1} = \left\{ \left[N_f s_{\alpha 1}^2 + N_h + \lambda \right] A \cos s_{\alpha 1} y + \left[-N_f s_{\beta 1}^2 + N_h + \lambda \right] \times B \cosh s_{\beta 1} y \right\} / N_h, \quad (23a)$$

$$\Theta_{s2} = \left\{ \left[N_f s_{\alpha 2}^2 + N_h + \lambda \right] C \sin s_{\alpha 2} (1 - y) + \left[-N_f s_{\beta 2}^2 + N_h + \lambda \right] D \sinh s_{\beta 2} (1 - y) \right\} / N_h. \quad (23b)$$

The heat flux at the boundary is given by

$$\begin{aligned} q'' &= \phi k_f \left(\frac{\partial T_f^*}{\partial y} \right)_{y^*=H} + (1 - \phi) k_s \left(\frac{\partial T_s^*}{\partial y} \right)_{y^*=H} \\ &= -\frac{T_{\text{ref}}}{H} e^{\lambda x} \left\{ \left[\phi k_f s_{\alpha 2} + (1 - \phi) k_s s_{\alpha 2} \right. \right. \\ &\quad \times \left(1 + \frac{N_f}{N_h} s_{\alpha 2}^2 + \frac{\lambda}{N_h} \right) \left. \right] C + \left[\phi k_f s_{\beta 2} + (1 - \phi) k_s s_{\beta 2} \right. \\ &\quad \times \left. \left. \left(1 - \frac{N_f}{N_h} s_{\beta 2}^2 + \frac{\lambda}{N_h} \right) \right] D \right\}. \end{aligned} \quad (24)$$

The difference between the effective bulk temperature and the wall temperature is

$$\begin{aligned} T_{\text{beff}} - T_w &= T_{\text{ref}} \left\{ \int_0^{\xi} [\phi \theta_{t1} + (1 - \phi) \theta_{s1}] dy \right. \\ &\quad \left. + \int_{\xi}^1 [\phi \theta_{t2} + (1 - \phi) \theta_{s2}] dy \right\} \\ &= T_{\text{ref}} e^{\lambda x} \left\{ \frac{AS_1}{s_{\alpha 1}} \left[\phi + (1 - \phi) \left(1 + \frac{N_f}{N_h} s_{\alpha 1}^2 + \frac{\lambda}{N_h} \right) \right] \right. \\ &\quad + \frac{BS_2}{s_{\beta 1}} \left[\phi + (1 - \phi) \left(1 - \frac{N_f}{N_h} s_{\beta 1}^2 + \frac{\lambda}{N_h} \right) \right] \\ &\quad + \frac{C(1 - C_3)}{s_{\alpha 2}} \left[\phi + (1 - \phi) \left(1 + \frac{N_f}{N_h} s_{\alpha 2}^2 + \frac{\lambda}{N_h} \right) \right] \\ &\quad \left. + \frac{D(C_4 - 1)}{s_{\beta 2}} \left[\phi + (1 - \phi) \left(1 - \frac{N_f}{N_h} s_{\beta 2}^2 + \frac{\lambda}{N_h} \right) \right] \right\}. \end{aligned} \quad (25)$$

We define the Nusselt number as

$$Nu = \frac{2Hq''}{k_{\text{eff}}(T_w - T_{\text{beff}})}. \quad (26)$$

It is now just a matter of substituting from Eqs. (24) and (25) into (26). The factor $T_{ref} e^{\lambda x}$ cancels. The expression may be simplified by using the quadratic equations satisfied by $s_{z1}^2, s_{z2}^2, s_{\beta1}^2$ and $s_{\beta2}^2$. Further simplifications can be made using the definitions in Eq. (7). In this manner one obtains the expression

$$Nu = \left(\frac{2}{(N_f + N_s)} \left\{ Cs_{z2} \left[N_f + \frac{\tilde{k}_{s2} N_s N_h}{N_h + \tilde{k}_{s2} N_s s_{z2}^2} \right] + Ds_{\beta2} \left[N_f + \frac{\tilde{k}_{s2} N_s N_h}{N_h - \tilde{k}_{s2} N_s s_{\beta2}^2} \right] \right\} \right) / \left(\left\{ \frac{AS_1}{s_{z1}} \right. \right. \\ \times \left[\phi + \frac{(1-\phi)N_h}{N_h + \tilde{k}_{s1} N_s s_{z1}^2} \right] + \frac{BS_2}{s_{\beta1}} \left[\phi + \frac{(1-\phi)N_h}{N_h - \tilde{k}_{s1} N_s s_{\beta1}^2} \right] \\ \left. \left. + \frac{C(1-C_3)}{s_{z2}} \left[\phi + \frac{(1-\phi)N_h}{N_h + \tilde{k}_{s2} N_s s_{z2}^2} \right] + \frac{D(C_4-1)}{s_{\beta2}} \left[\phi + \frac{(1-\phi)N_h}{N_h - \tilde{k}_{s2} N_s s_{\beta2}^2} \right] \right\} \right). \quad (27)$$

This expression can be readily checked for a special case, namely that for a homogeneous medium, for which

$$\tilde{k}_{s1} = \tilde{k}_{s2} = 1, \quad B = D = 0, \quad C = A, \\ s_{z1} = s_{z2} = \pi/2, \quad s_{\beta1} = s_{\beta2} = \infty. \quad (28)$$

For this case, Eq. (27) reduces to

$$Nu = \left\{ \pi^2 \left[N_f \left(N_h + \frac{\pi^2}{4} N_s \right) + N_s N_h \right] \right\} / \left\{ 2(N_f + N_s) \left[\phi \left(N_h + \frac{\pi^2}{4} N_s \right) + (1-\phi)N_h \right] \right\}, \quad (29)$$

an expression which is independent of the value of ξ , and which agrees with Eq. (36) of Nield and Kuznetsov [6] in the limit as L_f and s_2 both tend to infinity. In the particular case of large η (and hence of large N_h), as is normal in a practical situation, Eq. (29) leads to

$$Nu = \frac{\pi^2}{2} \left\{ 1 + \frac{\pi^2 \phi (1-\phi)^2 k_s (k_f - k_s)}{4h_{is} H^2 k_{eff}} \right\}. \quad (30)$$

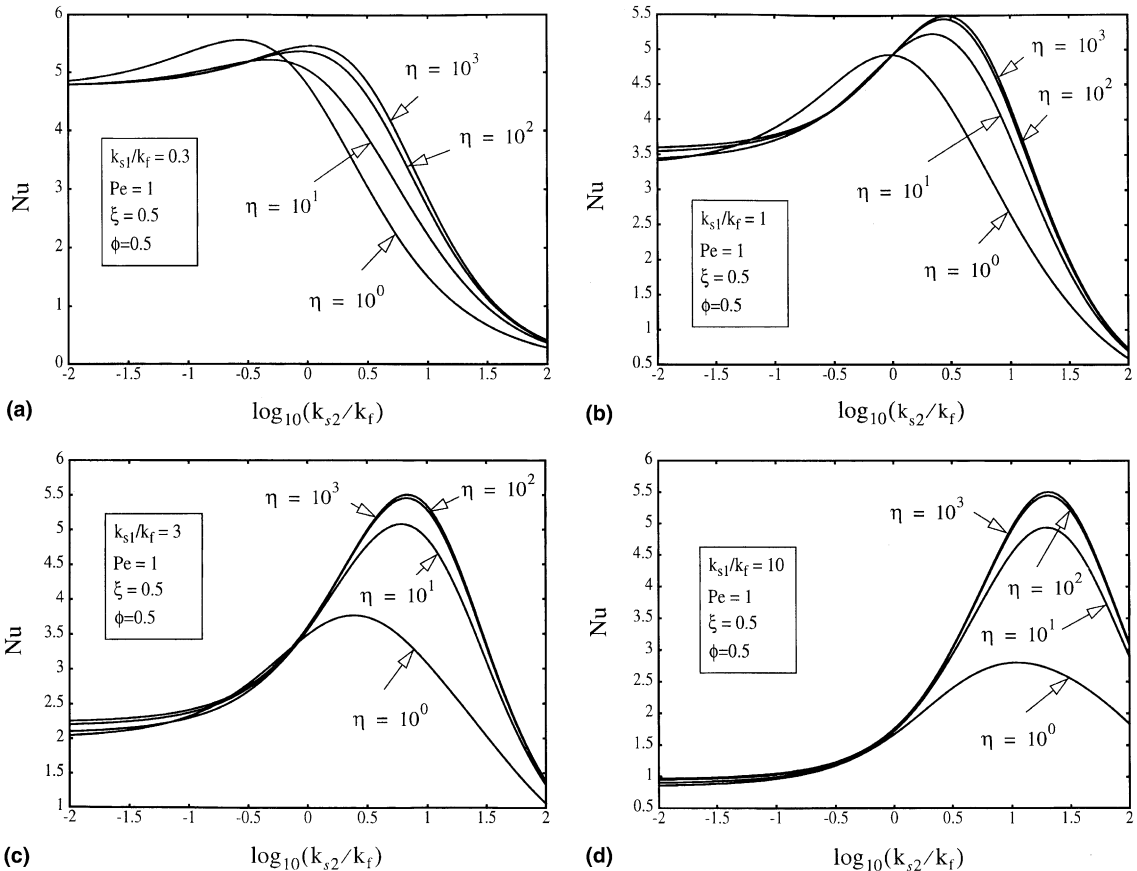


Fig. 2. Plots of Nusselt number versus solid-to-fluid conductivity ratio k_{s2}/k_f for various values of the solid-to-fluid heat transfer number η for: (a) $k_{s1}/k_f = 0.3$; (b) $k_{s1}/k_f = 1$; (c) $k_{s1}/k_f = 3$; (d) $k_{s1}/k_f = 10$ ($\xi = 0.5, \phi = 0.5$).

This illustrates the immediate fact that the effect of LTNE is negligible in the case where the porosity tends to either zero or unity or when the solid and fluid conductivities are closely the same. Further, it is seen that the effect of LTNE is to increase or decrease Nu (as we have defined it) according as k_s is less than or greater than k_f , respectively.

3. Results and discussion

We have calculated Nu from Eq. (27) for various parameter values. We confirmed that Nu is independent of Pe as expected. We checked that we could recover the results for the homogeneous case expressed in Fig. 3 of [6] in the limiting case of very large Biot number (corresponding to the isothermal boundary conditions that we have adopted in the present work). We then checked that we could obtain results in agreement with the middle curve in Fig. 4(b) of [1] for the case of local thermal equilibrium. This was a severe test of the correctness of the present analysis and its numerical implementation, because the formulation in the present paper is distinctly different from that in [1]. In the present paper an eigenvalue equation is solved for the separation constant (decay parameter) λ and the result is fed into an expression for the Nusselt number Nu , whereas in [1] the value of Nu is obtained directly from another eigenvalue equation.

Our results are presented in Figs. 1 and 2, for the case where the porosity is 0.5 and each layer occupies half of the channel. Fig. 2 has been designed for comparison with Fig. 4(b) of [1]. As we have already reported, the results for a large value of η (i.e., for negligible LTNE effects) match those in our earlier paper [1], and are in fact independent of the value chosen for k_{s1}/k_f . Fig. 1(b)

shows that the chief effect of LTNE is to reduce the Nusselt number when k_{effratio} is greater than unity (and has little effect when it is less than unity).

Fig. 2 presents results for various values of k_{s1}/k_f and k_{s2}/k_f varied separately. As k_{s1}/k_f increases the curves become more peaked, with the position of the peaks occurring at increasing values of k_{s2}/k_f and the effect of LTNE is to reduce the magnitude of the peaks. When k_{s1}/k_f is less than unity the effect of LTNE is small. (We have not presented results for $k_{s1}/k_f = 0.1$ because they closely resemble those for $k_{s1}/k_f = 0.3$.) The existence of the peaks was explained in [1] as a result of two opposing effects of conductivity variation.

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